Centre of Risk and Regulatory Excellence

Comment piece: Optimisation of non-life geographical diversification

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This comment piece aims to explain the optimisation of the non-life geographical diversification in Solvency II

Though an old idea, the measurement and allocation of diversification in portfolios of asset and/or liability risks is a difficult problem, which has found so far many answers. The diversification effect of a portfolio of risks is the difference between the sum of the risk measures of stand-alone risks in the portfolio and the risk measure of all risks in the portfolio taken together, which is typically non-negative, at least for positive dependent risks. The risk allocation problem consists to apportion the diversification effect to the risks of a portfolio in a fair manner, to obtain new risk measures of the risks of a portfolio. The first mathematical approach to diversification is due to 1990 Nobel laureate Markowitz, whose classical portfolio selection model applies to the efficient diversification of investments.

Within the Global View to solvency systems, the CRO Forum(2005), p.20, has classified diversification benefits into four distinct categories:

- **Level 1 – within risk types:** the diversification in a homogenous portfolio, for example through aggregation of unrelated risks in a portfolio or by investing in an index of common shares rather than in a single company
- **Level 2 – across risk types:** the diversification obtained by combining lines of insurance business, or the one obtained between insurance risk and market risk provided claims results are unrelated to investment risks
- **Level 3 – across entities, within a geography:** the diversification obtained by combining two or more insurance companies within a Group
- **Level 4 – across geographies or regulatory jurisdictions:** the diversification resulting from the consolidation of entities operating in different geographies

This four-way classification helps understand the differences in diversification treatment among the many solvency frameworks around the world (e.g. Table 2 in CRO Forum(2005), p.22). While Solvency I does not recognise any level of diversification, the Australia's APRA solvency system does already at all levels (with exceptions for geographical diversification).

The quantification of diversification effects usually follows two distinct roads:

1. The bottom-up method: In order to get the total risk capital required at the highest level of a group, one starts with the calculation at the lowest level of sub-risks. The issue is then to design methods for the combination of these sub-risks to obtain the risk capital at various higher levels
2. The top-down method: One aggregates the exposures for each risk of an organisation and calculates the overall required capital applying risk modeling or scenario analysis. Then, to obtain the required capital at intermediate levels of the organisation, one applies an appropriate capital allocation principle.

The first method is discussed by GCAE(2005) while Dhaene et al.(2009) provide an advanced comprehensive treatment of the second method. Hürlimann(2002) has developed a feasible practical way for the top-down calculation of insurance risks applying classical risk theory. The latter proposal has been re-used in another context to construct a gamma distributed IBNR claims reserving model with dependent development periods in Hürlimann(2007).

The forthcoming contribution Hürlimann(2009) considers a new top-down method for calculation of the level 4 diversification effect of a portfolio of non-life risks.

According to the current QIS4 standard approach, non-life risk capital charges take into account geographical diversification by adjusting volume measures using a Herfindahl-Hirschman concentration index for premiums and reserves at a line of business level. The lower the Herfindahl index the less concentrated is a portfolio and the greater is its diversification extent. While from a theoretical point of view the link between diversification and concentration has been somewhat studied in Foldvary(2006), the practical measurement of diversification in the Solvency II project has scarcely been discussed.

The diversification factor of a portfolio of risks with respect to some risk measure is defined to be the quotient of the portfolio risk measure to the sum of the stand-alone risk measures over all risks in the portfolio. Maximum diversification is obtained by minimising the diversification factor. Observe that the greater the diversification reduction is, the less risk capital is needed and the more new business can be written. Therefore optimal diversification has an important practical relevance.

According to the QIS4 proposal the minimum geographical non-life diversification factor is equal to 0.75. This value is not optimal. If the risk measure is proportional to the standard deviation of the risk, then the absolute minimum value of 0.707 allows for an additional diversification reduction of maximum magnitude 4.3%. The latter is true in the case of the VaR and the CVaR measures for the class of multivariate elliptical risk distributions. However, the current Solvency II standard approach to non-life risk relies on log-normal distributions. Under this assumption, the minimum diversification factor, which depends on the volatility of the portfolio, is in the average equal to 0.667, which results in an absolute diversification reduction of magnitude 8.3% compared to QIS4.

Extending the analysis to the class of multivariate log-elliptical risk distributions, further results on the minimum diversification factor can be obtained. For the class of multivariate log-Laplace distributions, which are able to model fat tails similarly to the class of generalised Pareto distributions in Extreme Value Theory, this minimum value is in the average 0.68 resulting in an absolute reduction of lower magnitude 7%.

The demonstration that the preceding findings are not only of theoretical interest is best done through the analysis of two simple examples. We compare the current QIS4 specification with our new alternative approach based on the common assumption of log-normally distributed non-life risks. Suppose a non-life insurer has five lines of business with the following correlation matrix:

$$\begin{pmatrix} 1 & 0.5 & 0.5 & 0.25 & 0.25 \\ 0.5 & 1 & 0.25 & 0.25 & 0.5 \\ 0.5 & 0.25 & 1 & 0.5 & 0.25 \\ 0.25 & 0.25 & 0.5 & 1 & 0.5 \\ 0.25 & 0.5 & 0.25 & 0.5 & 1 \end{pmatrix}$$

We complete the table below using 2 different assumptions about the Herfindahl indices to get the following summary of results

	Overall	Lines of business				
Volumes	1000	400	250	200	100	50
Standard deviations (std)	14.5 %	12 %	20 %	25 %	30 %	50 %
SCR (without Diversification)	435.6					
Example 1						
Herfindahl indices		0.25	0.5	0.6	0.75	1
QIS4 diversified volumes	867.5	325	218.75	180	93.75	50
QIS4 diversified overall std	14.9 %					
QIS4 SCR (with Diversification)	387.8					
Alternative diversified volumes	832.7	306.26	210.29	174.21	91.90	50
Alternative diversified overall std	14.9 %					
Alternative SCR (with Diversification)	375.1					
Example 2						
Herfindahl indices		0.1	0.2	0.3	0.4	0.5
QIS4 diversified volumes	803.75	310	200	165	85	43.75
QIS4 diversified overall std	14.7 %					
QIS4 SCR (with Diversification)	355.6					
Alternative diversified volumes	741.75	284.45	183.45	152.92	79.67	41.26
Alternative diversified overall std	14.8 %					
Alternative SCR (with Diversification)	329.3					

In example 1 the diversification effect equals 11% of the SCR without diversification under the QIS4 approach. Under the alternative approach this effect increases to 13.9%.

In the more diversified example 2 the diversification effect increases from 18.4% to 24.4%. We expect that the diversification effect always increases from the QIS4 approach to the alternative approach, which implies a release of required risk capital.

Conclusion

Let us mention that geographical diversification has been discussed previously in banking by Liang and Rhoades(1988) and Rose and Wolken(1990). A very recent paper on the copula approach to this topic is Larsen et al.(2009), which study geographical diversification in agriculture risk management.